GCSE Maths – Geometry and Measures

Congruence – Lengths, Areas and Volumes

Worksheet

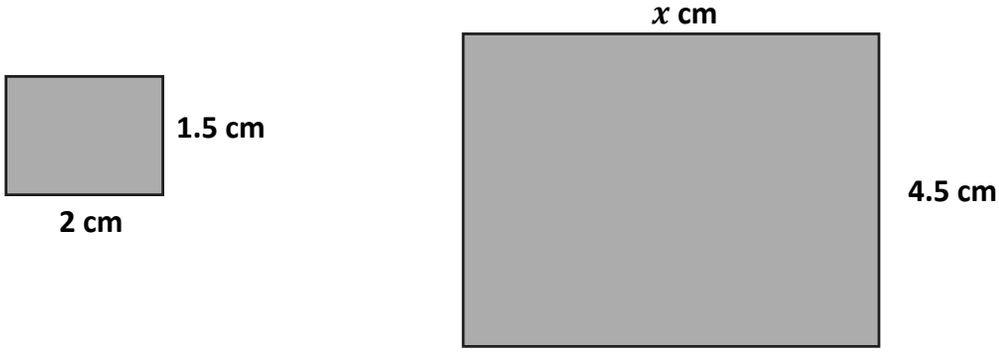
WORKED SOLUTIONS

This worksheet will show you how to work out different types of questions relating to congruence. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Given that the following rectangles are similar, calculate length x.



Step 1: Calculate the scale factor between the two similar shapes.

Scale factor is found by comparing two matching sides of the two similar shapes. We divide the short edge of the large rectangle by the short edge of the small rectangle:

$$4.5 \div 1.5 = 3$$

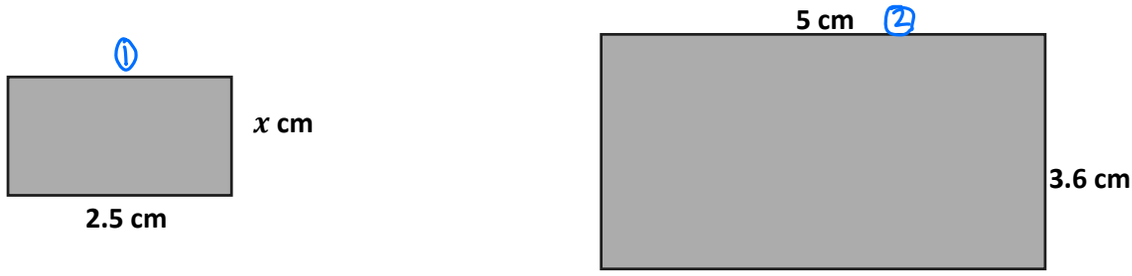
Scale factor is 3. This means the lengths of the larger rectangle are three times larger than the lengths of the smaller rectangle.

Step 2: Calculate x using the scale factor enlargement on the corresponding side of the similar rectangle.

$$x = 2\ cm \times scale\ factor = 2\ cm \times 3 = 6\ cm$$

Guided Example

Given that the following rectangles are similar, calculate length x.



Step 1: Calculate the scale factor between the two similar shapes.

sf = 5 ÷ 2.5 = 2 ← This means that ② is twice the lengths of ①

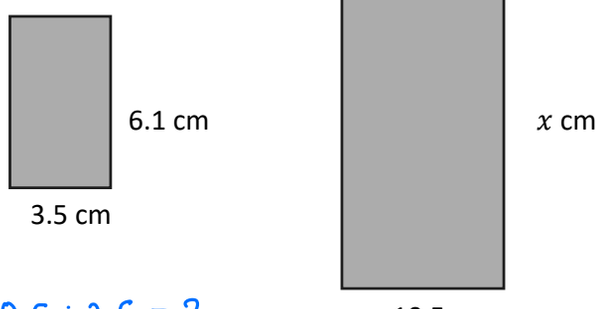
Step 2: Calculate x using the scale factor enlargement on the corresponding side of the similar rectangle.

$x = 3.6 \div 2$ (we divide as ① is smaller than ②)

$= 1.8$ cm

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

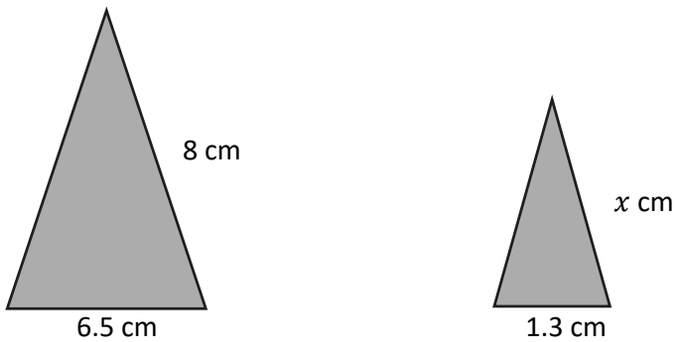
1. Given that the rectangles are similar, calculate length x.



Sf = 10.5 ÷ 3.5 = 3

x = 6.1 × 3 = 18.3 cm

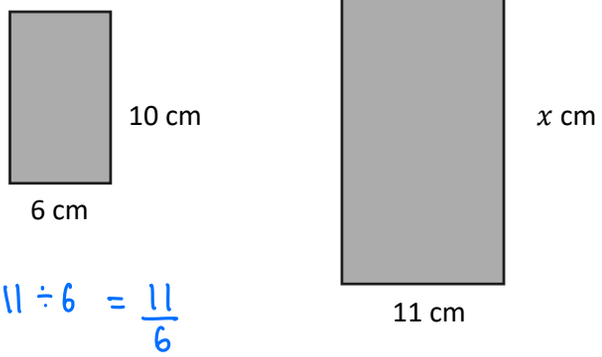
2. Given that the triangles are similar, calculate length x.



Sf = 6.5 ÷ 1.3 = 5

x = 8 ÷ 5 = 1.6 cm

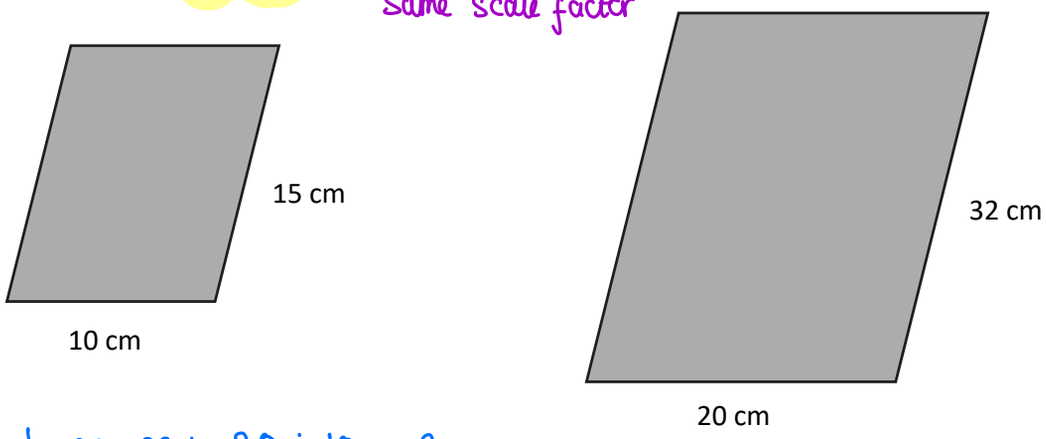
3. Given that the rectangles are similar, calculate length x.



$sf = 11 \div 6 = \frac{11}{6}$

$x = 10 \times \frac{11}{6} = \frac{110}{6}$ cm (Leave in fraction form)

4. Are these shapes similar? ← If the sides share the same scale factor



base sf: $20 \div 10 = 2$

length sf: $32 \div 15 = \frac{32}{15}$

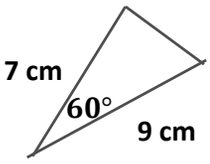
$2 \neq 32/15$

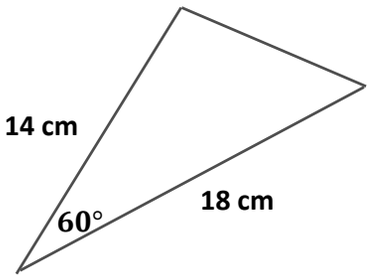
The scale factors for the sides are not equal, therefore the shapes aren't similar.

Section B

Worked Example

Prove that the two triangles are similar.





Step 1: Check that the scale factor of the sides of the triangles are the same.

Comparing matching sides of triangles:

$$18\ cm \div 9\ cm = 2$$
$$14\ cm \div 7\ cm = 2$$

For both sets of matching sides, the scale factor is 2.

Step 2: Check that the angles in the triangles match the condition of similar triangles.

For similar triangles, their corresponding angles must be equal.
This is indeed the case:

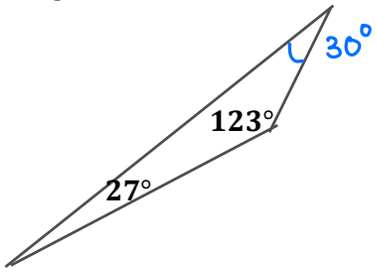
$$60° = 60°$$

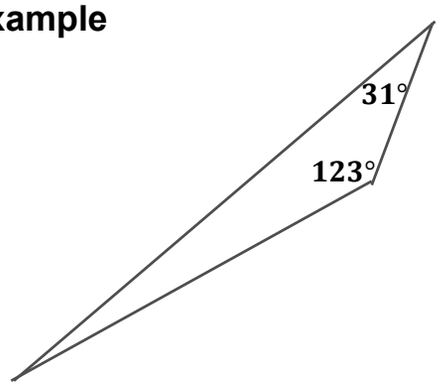
Step 3: Form a conclusion.

The triangles are similar as the lengths are enlarged by the same scale factor and the angle between these sides is equal.

Guided Example

Are the triangles similar?





Step 1: Check if the angles in the triangles match the condition of similar triangles.

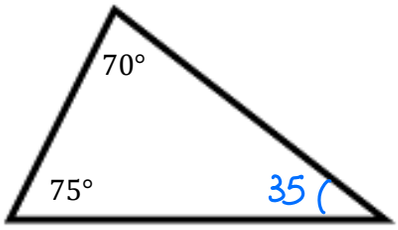
$180 - 123 - 27 = 30°$

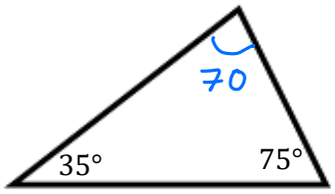
Step 2: Form a conclusion.

The angles do not match ($30 \neq 31$), therefore the triangles aren't similar.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. Prove that the triangles are similar.





$180 - 70 - 75 = 35$

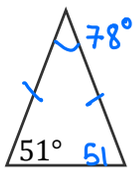
$180 - 75 - 35 = 70$

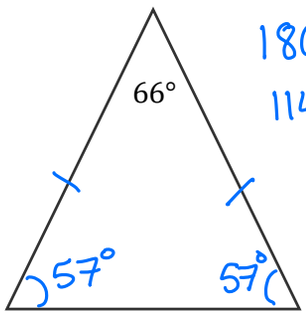
All angles of both triangles correspond and are equal.

∴ The triangles are similar.

6. Both triangles below are isosceles. Are they similar?

$180 - 51 - 51 = 68°$





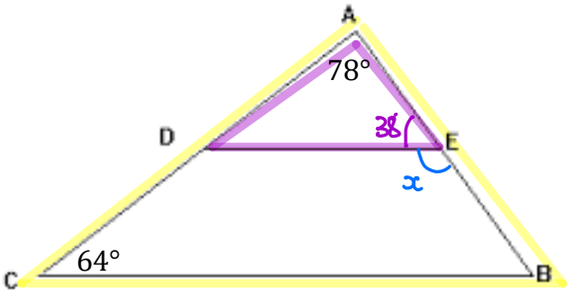
$180 - 66 = 114$

$114 \div 2 = 57$

The angles in the isosceles triangles aren't equal.

∴ The triangles are not similar.

7. Triangle ABC is similar to ADE. Calculate angle BED.



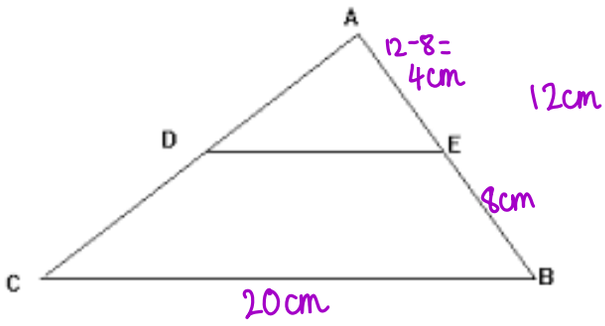
$180 - 64 - 78 = 38^\circ = \angle ABC$ (Triangle = 180°)

As $\triangle ABC$ and $\triangle ADE$ are similar $\angle ABC = \angle AED$

$\angle BED = 180 - 38$ (straight line)

$= 142^\circ$

8. In the following diagram $AB = 12$ cm, $EB = 8$ cm and $CB = 20$ cm. Triangles ABC and ADE are similar. Calculate the length of DE.



Scale factor:

$12 \div 4 = 3$

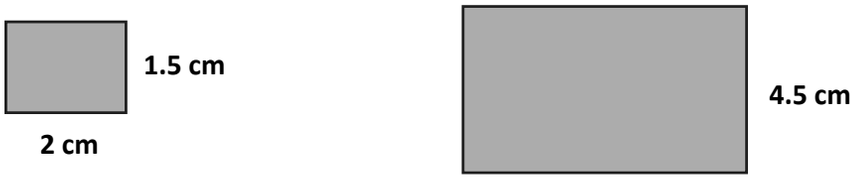
$DE = 20 \div 3$

$= \frac{20}{3}$ cm

Section C

Worked Example

Calculate area of the larger rectangle given the rectangles are similar.



Step 1: Calculate the scale factor between the two similar shapes.

Compare two corresponding sides to deduce the scale factor:

$$Scale\ factor = 4.5\ cm \div 1.5\ cm = 3$$

Step 2: Calculate the area of the smaller rectangle.

$$Area_{small} = 2\ cm \times 1.5\ cm = 3\ cm^2$$

Step 3: Calculate the area scale factor.

$$Area\ scale\ factor = (Scale\ factor)^2 = 3^2 = 9$$

Step 4: Calculate the area of the larger rectangle using the area scale factor.

Multiply the area scale factor by the area of the smaller rectangle:

$$Area_{large} = 3\ cm^2 \times 9 = 27\ cm^2$$

Guided Example

The area of the smaller rectangle and larger rectangle is $10\ cm^2$ and $62.5\ cm^2$. Calculate length x.

Step 1: Calculate the area scale factor between the two similar shapes.

Area sf = 62.5 ÷ 10
= 6.25

Step 2: Calculate the linear scale factor between the two similar shapes.

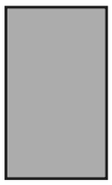
Linear sf = $\sqrt{6.25}$
= 2.5 cm

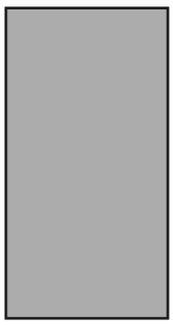
Step 2: Calculate length x using the linear scale factor.

5 x 2.5 = 12.5 cm

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

9. Given that the rectangles are similar, calculate the area scale factor.

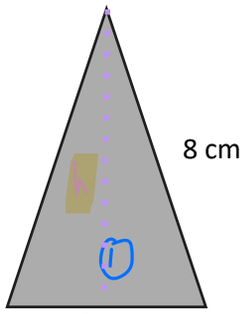


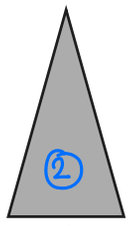


Linear sf = $10.5 \div 3.5$
$= 3$

Area sf = 3^2
$= 9$

10. Given that the triangles are similar, calculate the area of the smaller triangle to 1 decimal place.





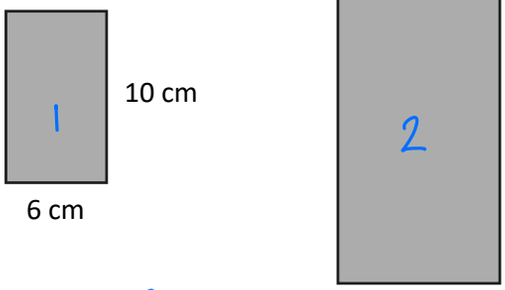
Linear sf = $6.5 \div 1.3 = 5$
Area sf = $5^2 = 25$

Finding h.
Using pythagoras theorem
$h = \sqrt{(8)^2 - (3.25)^2} = 7.31$ (2dp).

$\text{Area}_1 = \frac{1}{2} \times 6.5 \times 7.31$
$= 23.8$ (1dp)

$\text{Area}_2 = 25 \div 23.8$
$= 1.1$ (1dp)

11. The linear scale factor is 4. Given that the rectangles are similar, calculate area of the larger rectangle.



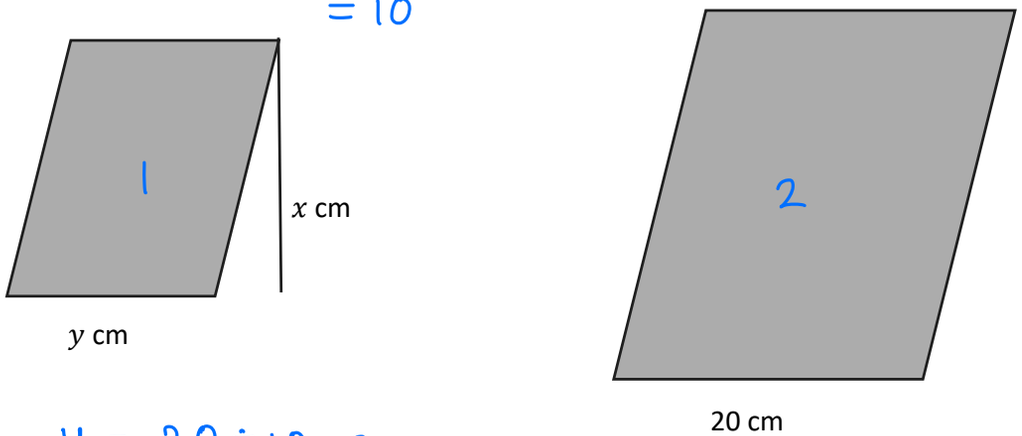
Area sf $= 4^2 = 16$

$\text{Area}_1 = 10 \times 6 = 60\text{cm}^2$

$\text{Area}_2 = 60 \times 16 = 960\text{cm}^2$

12. The area of the larger parallelogram is $900\ cm^2$. Given that the parallelograms are similar, and the area scale factor is 100, find x and y.

Linear sf $= \sqrt{100}$
$= 10$



$y = 20 \div 10 = 2$

$\text{Area}_1 = \text{Area}_2 \div \text{Area sf} = 900 \div 100 = 9\text{ cm}^2$

$x \times y = 9$ Area of parallelogram

$2x = 9$

$\div 2$

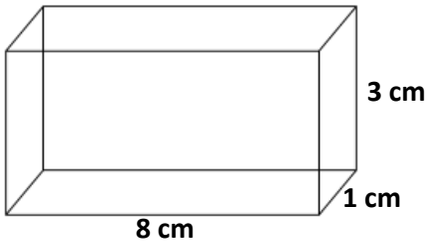
$x = 4.5$

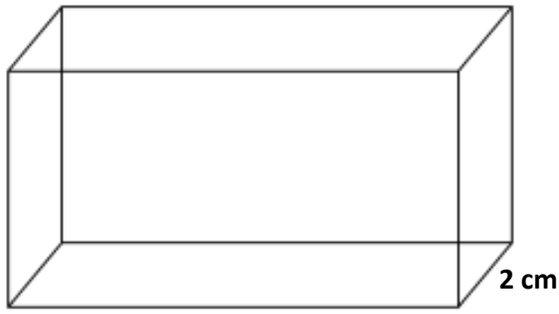
$x = 4.5$
$y = 2$

Section D

Worked Example

Given that the cuboids are similar, calculate the volume of the larger cuboid.





Step 1: Calculate the linear scale factor between the cuboids.

Comparing the shortest sides of the two cuboids:

$$Scale\ factor = 2\ cm \div 1\ cm = 2$$

Step 2: Calculate the volume scale factor.

$$Volume\ scale\ factor = (Scale\ factor)^3 = 2^3 = 8$$

Step 3: Calculate the volume of the smaller cuboid.

$$Volume_{small} = 8\ cm \times 1\ cm \times 3\ cm = 24\ cm^3$$

Step 3: Calculate the volume of the larger cuboid using the volume scale factor.

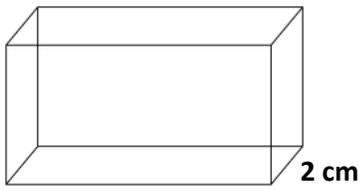
Multiply the volume of the smaller cuboid by the volume scale factor:

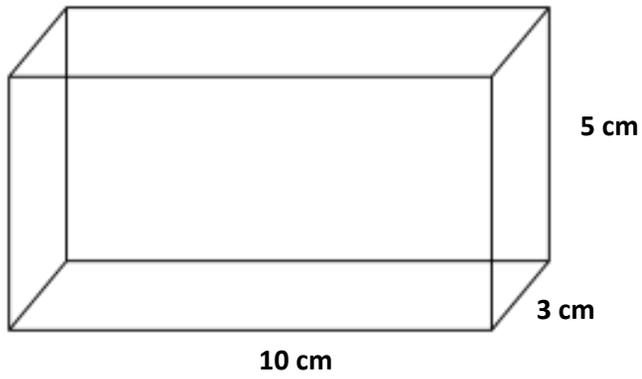
$$Volume_{large} = 24\ cm^2 \times 8 = 192\ cm^3$$

The volume of the larger cuboid is $192\ cm^3$**.**

Guided Example

Given the cuboids are similar, calculate the volume of the smaller cuboid.





Step 1: Calculate the linear scale factor between the cuboids.

$3 \div 2 = 1.5$

Step 2: Calculate the volume scale factor between the two cuboids.

$1.5^3 = 3.375$

Step 3: Calculate the volume of the larger cuboid.

$10 \times 5 \times 3 = 150\ cm^3$

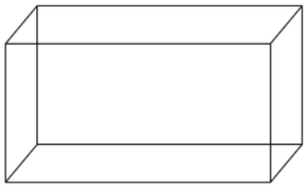
Step 4: *Calculate the volume of the smaller cuboid.*

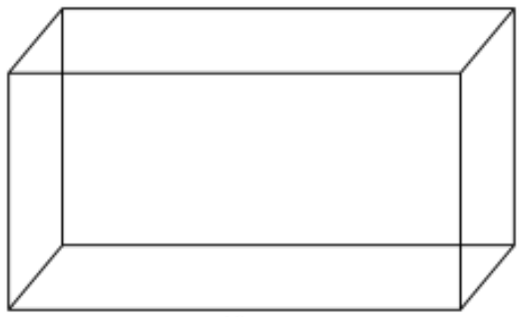
$150 \div 3.375 = 44.4\ cm^3$

$= 44.4\ cm^3$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

13. The volume scale factor is 27. Calculate length x cm.





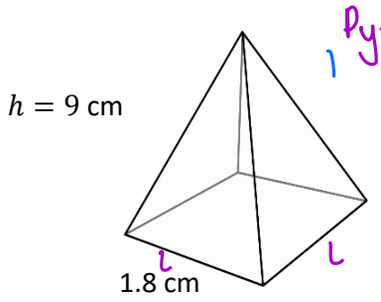
Vol sf = 27

Linear sf = $\sqrt[3]{27}$ = 3

42 ÷ 3 = 14

x = 14

14. Given that the square pyramids are similar, calculate the volume of the larger pyramid. The vertical height of the pyramid is denoted by h.



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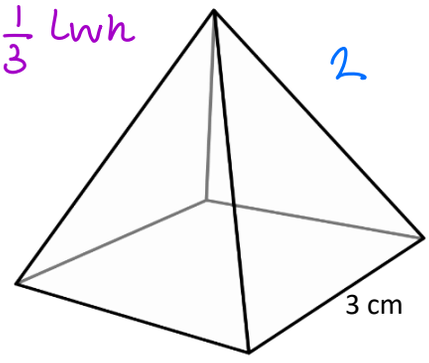
Pyramid vol = $\frac{1}{3}$ lwh

(l = w)



Linear sf = 3 ÷ 1.8 = $\frac{5}{3}$

Volume sf = $\left(\frac{5}{3}\right)^3$ = $\frac{125}{27}$

Vol_1 = $\frac{1}{3}$ x 1.8 x 1.8 x 9 = 9.72 cm^3

Vol_2 = 9.72 x $\frac{125}{27}$ = 45 cm^3

15. The surface area of cube A is $24\ cm^2$. The surface area of cube B is $54\ cm^2$ and the volume is $27\ cm^3$. Calculate the volume of cube A.

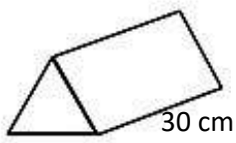
Area sf: $54 \div 24 = \frac{54}{24} \overset{\div 6}{\underset{\div 6}{=}} \frac{9}{4}$

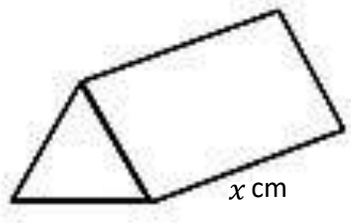
Linear sf: $\sqrt{\frac{9}{4}} = \frac{3}{2}$

Vol sf: $\left(\frac{3}{2}\right)^3 = \frac{27}{8}$

$\text{Vol}_A = 27 \div \frac{27}{8} = 8\ cm^3$

16. The volume of the smaller prism is $25\ cm^3$ and the volume of the larger prism is $200\ cm^3$. Find the value of x





Vol sf: $200 \div 25 = 8$

Linear sf: $\sqrt[3]{8} = 2$

$x = 30 \times 2 = 60$